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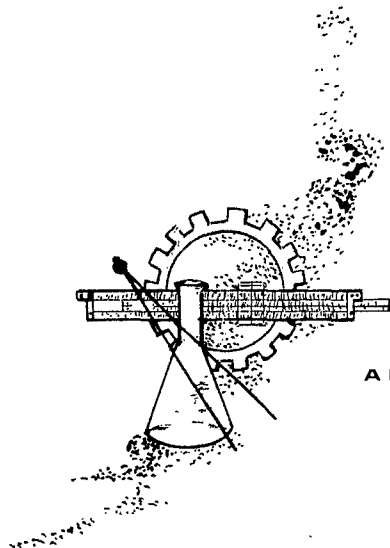


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APPLIED MECHANICS....PHYSICS....ANALYSES

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**ON THE USE OF IMPACT THEORY FOR SLENDER CONFIGURATIONS  
EXHIBITING FLOW SEPARATION**

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## SUMMARY

A method is presented for predicting analytically the non-linear variation of normal force with angle of attack for very slender wing-body combinations exhibiting flow separation. The method is based on the cross-flow drag concept and employs Newtonian impact theory in two dimensions to calculate the cross-flow drag coefficient. It is demonstrated that this theory shows good agreement with experiment for two-dimensional incompressible flow past bluff shapes. Furthermore, the analytical expression for the flat plate agrees exactly with the theoretical analysis of Bollay.

Mathematical expressions are developed for the normal force on a conical slender wing-body combination with elliptical body of arbitrary eccentricity. Calculations are compared with experimental normal forces on wing-body combinations at supersonic speeds and on delta wings at subsonic speeds. The agreement appears to be very good over a wide range of angle of attack, provided that the slenderness approximation is satisfied and that separation occurs near the lateral extremities of the configuration in question. For the sharp-edged delta wing, the agreement with experiment is better than for other theories.

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# LIST OF SYMBOLS

A	aspect ratio, $d_o^2/s$
a	local half-depth of body on wing-body combination
b	local half-width of body on wing-body combination
$C_{D_c}$	cross-flow drag coefficient, $D/(1/2)\rho V_\infty^2 d$
$C_L$	lift coefficient, $L/(1/2)\rho V_\infty^2 s$
$C_{L_\alpha}$	lift-curve slope
$C_N$	normal-force coefficient, $N/(1/2)\rho V_\infty^2 s$
$C_p$	pressure coefficient, $(p - p_o)/(1/2)\rho V_\infty^2$
D	aerodynamic drag force per unit length, lb/ft
d	span of section measured normal to free stream, ft
$E( )$	complete elliptic integral of the second kind
k	$\sigma - \frac{(a + b)^2}{4\sigma}$
$k_1$	lift-curve slope given by slender-body theory
L	lift force, lb
$M_\infty$	free-stream Mach number
N	normal force, lb
S	planform area, ft <sup>2</sup>
s	semispan of wing-body combination, ft
$V_\infty$	free-stream velocity, ft/sec
$\alpha$	angle of attack

$\beta$	$\sqrt{ M_{\infty}^2 - 1 }$
$\delta$	local angle between a surface and free stream
$\epsilon$	semivertex angle of delta wing
$\lambda$	$\frac{1}{E \left( \sqrt{1 - \beta^2 \tan^2 \epsilon} \right)}$
$\rho$	air mass density, slugs/ft <sup>3</sup>
$\sigma$	$\frac{1}{2} \left( s + \sqrt{s^2 + a^2 + b^2} \right)$

#### Subscripts

b	on the body
o	at trailing edge of slender configuration
w	on the wing



# ON THE USE OF IMPACT THEORY FOR SLENDER CONFIGURATIONS EXHIBITING FLOW SEPARATION

## 1. INTRODUCTION

It has long been recognized that steady flow separation may exist on slender configurations over a wide range of Mach number and angle of attack. However, except for the cases of the low-aspect-ratio rectangular wing and the low-aspect-ratio delta wing, very little progress has been made in the analysis of separated flow over slender configurations in general. The rectangular wing was first treated in a simple manner by Betz (Ref. 1) who introduced the concept of using the "cross-flow drag coefficient" of a two-dimensional flat plate placed normal to the stream. Then in 1939, Bollay published a rigorous analysis of the flat rectangular plate with side-edge separation (Ref. 2) in which he solved an integral equation for the loading and determined the corresponding normal force and the shedding angle of the separated vortices.

More recently, the slender delta wing with leading-edge separation has been analyzed by a number of investigators (Refs. 3 through 7). In addition, the body of revolution with flow separation has been treated by Allen and Perkins (Ref. 8), who use the cross-flow drag concept and suggest that this concept be used to predict the nonlinear lift curve for slender configurations in general, using experimental values for the drag coefficient over a two-dimensional body having the same shape as the cross-section of the configuration in question. However, no analytical method has yet been developed for predicting the required cross-flow drag coefficient for general shapes.

In the present paper, a method will be presented for calculating the nonlinear normal force for general slender configurations exhibiting flow separation. The method is based on the use of Newtonian impact theory to determine the required cross-flow drag coefficient. It will be demonstrated that this method yields drag coefficients on two-dimensional bodies which are in good agreement with experiment at low speeds, so long as the wake width is essentially equal to the maximum lateral dimension of the body.

Calculations will be made of the resulting nonlinear lift curves for a class of slender wing-body combinations which have been tested over a range of supersonic Mach numbers. In addition, the special case of the flat-plate delta wing will be calculated and compared with other theories and with subsonic experimental data.

During the preparation of the present paper, a similar argument was published by Mysliwetz (Ref. 9) to show that Newtonian theory may be used outside the Mach number range in which it correctly represents the physical flow. However, Mysliwetz's work was confined to bodies of revolution and led him to a formula which predicted a nonlinear term in the normal force, only if a cylindrical afterbody is present. Therefore, his predicted normal-force curve for a circular cone would be linear and would coincide with that predicted by slender-body theory. This gives the correct slope at  $\alpha = 0$  but fails completely to predict the nonlinear viscous effects, which are appreciable at moderate angles of attack. The approach presented in the present report does not suffer from this shortcoming and evidently gives good predictions of normal force over the entire practical angle-of-attack range, not only for cones, but also for slender wings and wing-body combinations of rather general cross section.

## 2. CROSS-FLOW DRAG CONCEPT

The leading term in the Prandtl-Glauert equation of linearized theory

$$\left(1 - M_{\infty}^2\right) \phi_{xx} + \phi_{yy} + \phi_{zz} = 0 \quad (1)$$

can be neglected either if  $M_{\infty} = 1$  (sonic flight) or if the configuration is so slender that rates of change of the perturbation velocities in the flight direction are negligibly small compared with those normal to it (see Ref. 10). The problem is thereby reduced to the two-dimensional one of Laplace's equation in planes normal to the flight direction. In Reference 11, R. T. Jones

showed that for a slender, pointed flat plate this leads to the following simple expression for the normal force:

$$C_{N_{A \rightarrow 0}} = \frac{\pi}{2} A \alpha \quad (2)$$

where  $A$  is the aspect ratio.

Perhaps the first to attempt an analysis of the separated flow on slender bodies at high angles of attack was Allen (Ref. 8), who suggested that the linearized slender-body theory of Jones (Ref. 11) and Ward (Ref. 12) be modified by the addition of a quadratic term in the expression for the normal force. Thus,

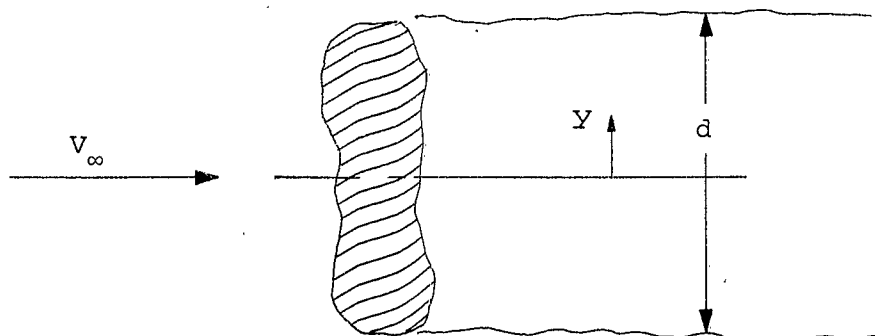
$$C_{N_{A \rightarrow 0}} = k_1 \alpha + k_2 \alpha^2 = k_1 \alpha + C_{D_C} \alpha^2 \quad (3)$$

where  $k_1$  is the lift-curve slope given by slender-body theory and  $k_2$  represents the "cross-flow drag coefficient"  $C_{D_C}$  which is the drag coefficient of a two-dimensional body having the same cross-sectional shape as the slender body in question. Allen suggested that the value of  $C_{D_C}$  be obtained from low-speed, two-dimensional wind-tunnel tests of various cross sections. He showed an improvement over slender-body theory for the lift on a body of revolution at high angles of attack. It is noted here that we use lift and normal force interchangeably, since sample calculations have shown that, for the lift-drag ratio of the delta wing through the angle-of-attack range investigated herein, the maximum difference between lift and normal force is about 2 percent.

Since the appearance of Reference 8, a number of attempts have been made to investigate the validity of Equation (3) (e.g., Ref. 13), but there is as yet no analytical technique for calculating the coefficient  $C_{D_C}$  for slender bodies of general cross section. The purpose of this report is to present such a method.

### 3. RELATION OF IMPACT THEORY TO BOLLAY'S ANALYSIS

A correlation of drag data on two-dimensional bluff bodies of various cross sections indicates that, so long as the cross section is sufficiently bluff and the edges sufficiently sharp so that separation occurs essentially at the side edges independent of Reynolds number, then the wake has the same width as the body and the drag of the section is accurately given by Newtonian impact theory. That is, for bodies of the type shown below,



the drag is closely predicted by assuming that each elemental particle of fluid strikes the body at the free-stream velocity  $V_{\infty}$  and thereupon loses its normal component of momentum. This leads to the well-known expression for the pressure coefficient:

$$C_p = 2 \sin^2 \delta \quad (4)$$

where  $\delta$  is the local angle which the forward-facing surface makes with the free stream. Points on the rearward face (i.e., in the wake) are assumed to experience zero pressure coefficient. The total drag coefficient of the cross section is then given by

$$C_{D_c} = \frac{D}{\frac{1}{2} \rho V^2 d} = \frac{1}{d} \int_{-d/2}^{d/2} C_p dy = \frac{2}{d} \int_{-d/2}^{d/2} \sin^2 \delta dy \quad (5)$$

where  $d$  is the span of the section measured normal to the free stream. The resulting drag coefficients for several bluff cross sections are given in Table I along with their corresponding experimental values. It can be seen that, for sufficiently bluff, symmetric bodies the correlation is quite good.

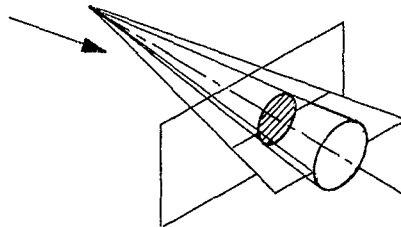
The drawback of the foregoing type of analysis is that a fluid does not behave in the assumed manner at low subsonic speeds in that the fluid particles are actually deflected far ahead of the body. For that reason, the agreement between the above expressions and experiment might be considered a coincidence. However, in Reference 2, W. Bollay performed an aerodynamic analysis of the forces acting on a low-aspect-ratio, rectangular wing at high angles of attack in an incompressible fluid, with separation at the side edges. In the limiting case of zero aspect ratio at  $90^\circ$  angle of attack, the normal force on Bollay's wing becomes the drag on a two-dimensional flat plate normal to the stream. The resulting equation for the normal force as a function of angle of attack is identical with that predicted by Newtonian theory. That is,

$$C_{N_{A=0}} = 2 \sin^2 \alpha \quad (6)$$

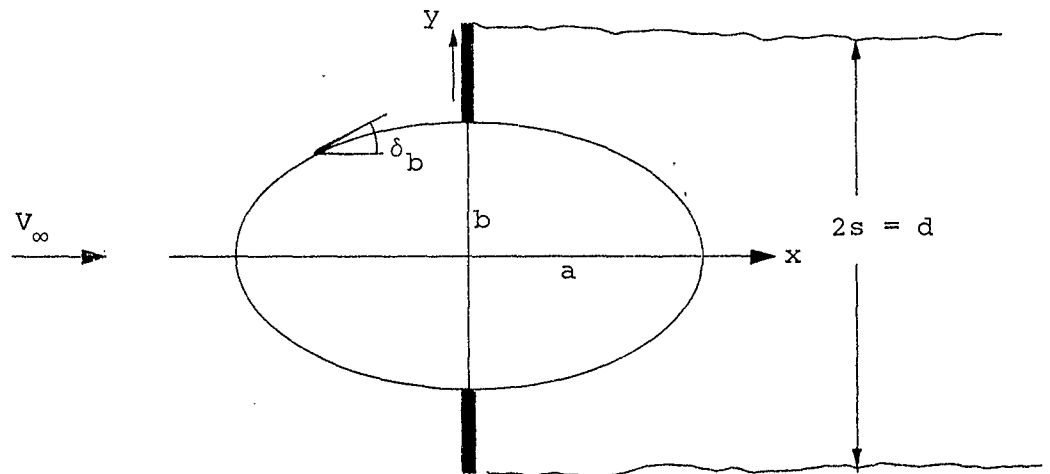
and we find from Equation (3) that  $C_{D_c} = 2$  (note that  $k_1 = 0$  at  $A = 0$ , see Eq. (2)). Thus, Equation (5) is actually verified by Bollay's mathematical analysis for the case of a flat plate, and furthermore appears to be justified on an empirical basis for other symmetrical, bluff shapes by the correlation of Table I.

#### 4. ANALYSIS OF SLENDER WING-BODY COMBINATIONS

On the basis of the foregoing argument, it is proposed that the cross-flow drag coefficient  $C_{D_C}$  of Equation (3) be calculated by means of Newtonian impact theory. Let us consider, for example, a slender wing-body combination consisting of an elliptical body and a wing with sharp leading edges as shown in the following sketch:



The corresponding two-dimensional flow problem to be solved for  $C_{D_C}$  is shown below. Separation at the wing leading edges is assumed.



We shall apply the impact theory by assuming no rebounding of particles from the body onto the projecting flat plate, since this would produce either crossing streamlines or secondary collisions of particles, neither of which seems justified. Thus, there is no wing-body interference, and the calculation is straightforward. On the body proper, which we shall consider to be of elliptic cross section, we have (see sketch above)

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \quad (7)$$

so that

$$\tan \delta_b = \frac{dy}{dx} = -\frac{b^2}{a^2} \left( \frac{x}{y} \right) \quad (8)$$

Thus, since

$$\frac{x}{y} = \frac{a}{b} \sqrt{\frac{b^2}{y^2} - 1} \quad (9)$$

we find that

$$C_{p_{body}} = 2 \sin^2 \delta_b = 2 \left[ \frac{b^2 - y^2}{b^2 - y^2 \left( 1 - \frac{a^2}{b^2} \right)} \right] \quad (10)$$

for  $-b < y < +b$ .

Now, on the wing panels (i.e., for  $b < y < d/2$  and  $-d/2 < y < -b$ ) where  $\delta_w = \pi/2$ , we have

$$C_{p_{wing}} = 2 \sin^2 \delta_w = 2 \quad (11)$$

Therefore, the cross-flow drag coefficient  $C_{D_c}$  for the wing-body combination is, from Equations (5), (10), and (11),

$$\begin{aligned}
 c_{D_c} &= \frac{2}{d} \int_0^b c_{p_{body}} dy + \frac{2}{d} \int_b^{d/2} c_{p_{wing}} dy \\
 &= \frac{4}{d} \int_0^b \frac{b^2 - y^2}{b^2 - y^2 \left(1 - \frac{a^2}{b^2}\right)} dy + \frac{4}{d} \left(\frac{d}{2} - b\right) \\
 &= 4 \frac{b}{d} \left[ \frac{a^2/b^2}{\left(\frac{a^2}{b^2} - 1\right)^{3/2}} \tan^{-1} \sqrt{\frac{a^2}{b^2} - 1} - \frac{1}{\frac{a^2}{b^2} - 1} + \frac{1}{2 \frac{b}{d}} - 1 \right] \quad (12)
 \end{aligned}$$

The above integration is valid for all values of the body eccentricity  $a/b$ , although for  $a/b < 1$  the inverse tangent of the complex argument  $\sqrt{(a^2/b^2) - 1}$  then becomes the logarithm of the real variable  $\sqrt{1 - (a^2/b^2)}$ . Equation (12) therefore gives the value of  $C_{D_c}$  for a whole class of sharp-edged, slender wing-body combinations for bodies of elliptic cross section.

It can be seen that Equation (12) properly approaches the flat-plate value of  $C_{D_c} = 2$  if either  $a$  or  $b$  goes to zero.

## 5. NORMAL-FORCE CALCULATIONS

In this section, we shall consider a specific family of conical, slender wing-body combinations for which the normal force calculated by Equations (3) and (12) can be compared directly with the systematic (supersonic) experiments of Reference 14. Also, the limiting case of  $a = 0$  (the flat-plate wing) will be considered and compared with subsonic experimental data and with previously developed wing theories.



### 5.1 Wing-Body Combinations

It has been shown in Reference 15 that for supersonic flow a better approximation for the lift-curve slope at zero angle of attack can be obtained by multiplying the slender-body value  $k_1$  for the particular configuration by a factor  $\lambda$ . This factor is the ratio of the lift of the wing alone by linearized theory to the lift by slender-body theory and is given by

$$\left. \begin{aligned} \lambda &= \frac{1}{E(\sqrt{1 - \beta^2 \tan^2 \epsilon})} \quad \text{for } \beta \tan \epsilon \leq 1 \\ &\quad \text{(subsonic leading edge)} \\ \text{or} \\ \lambda' &= \frac{2}{\pi \beta \tan \epsilon} \quad \text{for } \beta \tan \epsilon \geq 1 \\ &\quad \text{(supersonic leading edge)} \end{aligned} \right\} \quad (13)$$

where  $E(\ )$  is a complete elliptic integral of the second kind. We shall be concerned here only with the first form, since the slenderness assumption is clearly violated if the wing leading edges lie outside the Mach cone. Thus, we shall multiply the slender-body lift-curve slope  $k_1$  by  $\lambda$  to obtain the linear term of Equation (3).

The slender-body lift-curve slope for wing-body combinations with bodies of elliptical cross section has been given by Bryson (Ref. 16) in the form

$$C_{L_\alpha} = \frac{2\pi}{S} (k^2 + b^2)_{x=1} \quad (14)$$

where

$l$  = length of configuration

$$k = \sigma - \frac{(a + b)^2}{4\sigma}$$

$$\sigma = \frac{1}{2} \left( s + \sqrt{s^2 + a^2 - b^2} \right)$$

$S$  = planform area

Therefore, multiplying this expression by  $\lambda$  of Equation (13), we find the following equation for the normal force, according to Equations (3), (12), and (14):

$$C_N = \frac{1}{E \left( \sqrt{1 - \beta^2 \tan^2 \epsilon} \right)} \left[ \frac{2\pi}{S} (k^2 + b^2)_{x=l} \right] \alpha + 4 \frac{b}{d} \left[ \frac{\frac{a^2/b^2}{\left( \frac{a^2}{b^2} - 1 \right)^{3/2}} \tan^{-1} \sqrt{\frac{a^2}{b^2} - 1} - \frac{1}{\frac{a^2}{b^2} - 1} + \frac{1}{2 \frac{b}{d}} - 1 \right] \alpha^2 \quad (15)$$

## 5.2 Limiting Case of Flat-Plate Wing

As noted previously, the cross-flow drag coefficient of Equation (12) approaches a value of 2.0 as the body thickness term  $a$  approaches zero and the wing-body combination becomes a flat plate. This limiting case is considered for subsonic flow since the majority of the available data for sharp-edged delta wings for which lift or normal force was measured have been obtained for the subsonic regime. The values for the linear term of Equation (3) are therefore obtained from the subsonic linearized lift-curve slopes determined by Truckenbrodt in Reference 17. The subsonic normal force coefficient for a low-aspect-ratio, flat-plate wing is then,

$$C_N = C_{L_\alpha} \text{ (linearized) } \alpha + 2\alpha^2 \quad (16)$$

where values of  $C_{L_\alpha}$  are given as a function of aspect ratio in Figure 1.

## 6. COMPARISON WITH EXPERIMENT

### 6.1 Wing-Body Combinations

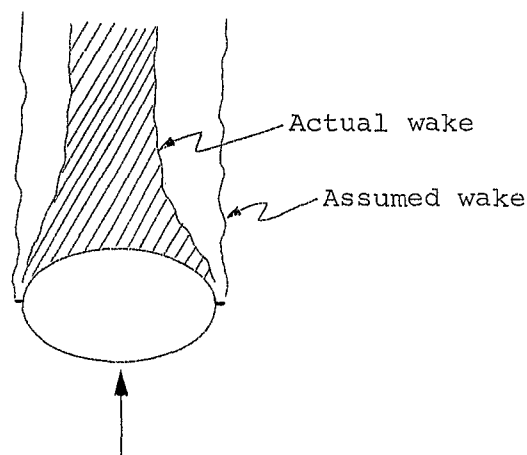
Calculations have been carried out with Equation (15) for the normal force on the following conical wing-body combinations:

<u>a/b</u>	<u>b/s</u>	<u>A</u>	<u>M<sub>∞</sub></u>	<u>Fig. No.</u>
1.0	0.544	1.0	1.97, 2.94	2, 4
	.362	1.5	1.97	3
3.0	.316	1.0	1.97, 2.94	5, 7
	.210	1.5	1.97	6
.333	.946	1.0	1.97, 2.94	8, 10
	.630	1.5	1.97	9

The resulting normal-force curves are presented in Figures 2, 3, and 4 for the circular body ( $a = b$ ), and it can be seen that the agreement with experiment is best for the lower aspect ratio at the lower Mach number (Fig. 2). This is to be expected since these are the cases which most nearly satisfy the slender-body assumption of Equation (3) that the wing leading edges lie well within the Mach cone. For aspect ratio 1 at  $M_\infty = 1.97$  (Fig. 2), the departure from linear theory is seen to be substantial (about 20 percent at  $\alpha = 8^\circ$ ), whereas the present theory shows good agreement with experimental data up to about  $15^\circ$  angle of attack. Both the higher aspect ratio ( $A = 1.5$ ,  $M_\infty = 1.97$ , (Fig. 3)) and the higher Mach number ( $M_\infty = 2.94$ ,  $A = 1.0$ , (Fig. 4)) show less

satisfactory agreement at the higher angles of attack. In fact, for the higher aspect ratio ( $A = 1.5$ ,  $M_\infty = 1.97$ , (Fig. 3)) the linear theory is actually somewhat better at the higher angles than the nonlinear expression of Equation (15), with experiment falling between the two. The higher aspect ratio at the higher Mach number ( $A = 1.5$ ,  $M = 2.94$ ) was not calculated, since the leading edges are actually slightly supersonic in that case and the present theory is clearly not applicable.

For wings with elliptic bodies ( $a \neq b$  (Figs. 5 to 10)), again the best agreement is obtained at the lower Mach number (Figs. 5, 6, 8, and 9). However, the effect of aspect ratio on the agreement is not so pronounced (compare Fig. 5 with Fig. 6), and a rather surprising effect of body eccentricity is noted. That is, with the major axis of the ellipse oriented vertically ( $a/b = 3$  (Fig. 6)), the agreement between the present theory and experiment at the lower Mach number and lower aspect ratio is again very good up to angles of attack of about  $15^\circ$ . However, with the elliptic body oriented with its major axis horizontal ( $a/b = 0.33$  (Fig. 8)), the agreement is not nearly as good. In fact, the linear theory seems to be better up to about  $10^\circ$  angle of attack! On the other hand, it was shown in Table I that impact theory agrees better with two-dimensional experiments if the major axis of the ellipse is normal to the stream. These two observations appear to be contradictory. However, it is pointed out that, for the slender wing-body combination with the ellipse horizontal, the wing leading edges just barely protrude from the body. It would therefore appear that the wing may be acting as a boundary-layer trip in the cross-flow plane and thereby fixes transition and delays cross-flow separation. Thus, the wake width, and consequently the cross-flow drag, is reduced (see sketch).



This would, of course, reduce the total normal force and produce a flow pattern more closely approximated by the attached-flow (linearized) theory.

## 6.2 Limiting Case of Flat-Plate Wing

Calculations have been carried out using Equation (16) for the normal force on flat-plate delta wings of aspect ratio 0.78, 1.5, and 2.0. The resulting curves of normal force versus angle of attack are presented in Figures 11(a), 11(b), and 11(c). It can be seen that the agreement with experiment is quite good, especially at the lower aspect ratios, which more nearly correspond to the slender-body assumption of Equation (3). The present theory clearly agrees better with experimental data than either the linear theory of Truckenbrodt (Ref. 17) or that of Jones (Ref. 11). Furthermore, it gives better agreement than any of the nonlinear theories developed from the separation vortex models of Gersten (Ref. 18), Brown and Michael (Ref. 3), or Mangler and Smith (Ref. 7). In Reference 13, Bartlett and Vidal arrive empirically at the same result as the present analysis for the special case of the sharp-edged delta wing. It is noted that for these

data and theories, the normal force and lift have been used interchangeably. This was necessary because the theories predict either lift or normal force and do not include expressions for drag which would be required to convert from lift to normal force or vice versa. However, as stated previously, the maximum difference between lift and normal force for the cases investigated here is approximately 2 percent.

It seems clear, then, that the present theory works quite well so long as the leading edges are well within the Mach cone and provided that separation does actually occur along the wing leading edges.

## 7. CONCLUDING REMARKS

A method has been presented for calculating the nonlinear lift curve for general slender configurations exhibiting steady flow separation. The method is based upon the observation that Newtonian impact theory yields good estimates for the low-speed drag of bluff bodies, and uses this theory to calculate the required cross-flow drag coefficient. A quadratic term in the angle of attack is thus obtained for adding the effect of viscous cross-flow to the lift calculated by linear theory.

Mathematical expressions have been developed for predicting the normal force on a class of conical, slender wing-body combinations with bodies of elliptic cross section. The calculations have been compared with experimental normal-force measurements over a range of supersonic Mach numbers, and it was found that the agreement is generally quite satisfactory up to about  $12^\circ$  angle of attack, provided that the leading edges lie well within the Mach cone and that separation does occur along the wing leading edges.

Calculations were also made for sharp-edged flat-plate delta wings of several aspect ratios and compared with experimental data at low subsonic speeds. It was found that the present theory

agrees well with experiment up to about  $20^{\circ}$  angle of attack and shows better agreement than do any of the previously existing theories for delta wings with leading-edge separation.

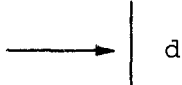
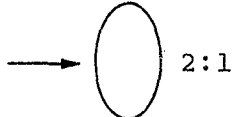
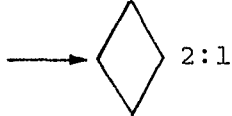
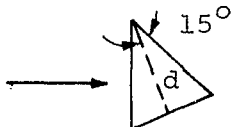
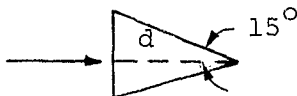
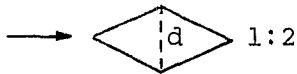
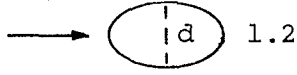
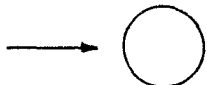
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TABLE I.- DRAG COEFFICIENT OF TWO-DIMENSIONAL BODIES  
IN INCOMPRESSIBLE FLOW AS GIVEN BY NEWTONIAN THEORY  
AND EXPERIMENT.

Cross section	$C_{D_{theo.}}$	$C_{D_{exp.}}$	Experimental ref.
 d	2.00	1.98	19
 2:1	1.65	1.7	20, 21
 2:1	1.6	1.8	21
 15°	1.94	2.28	20
 15°	2.00	1.90	20
 1:2	.4	1.00	21
 1.2	.95	.7	19, 20, 21
	1.33	1.20	22

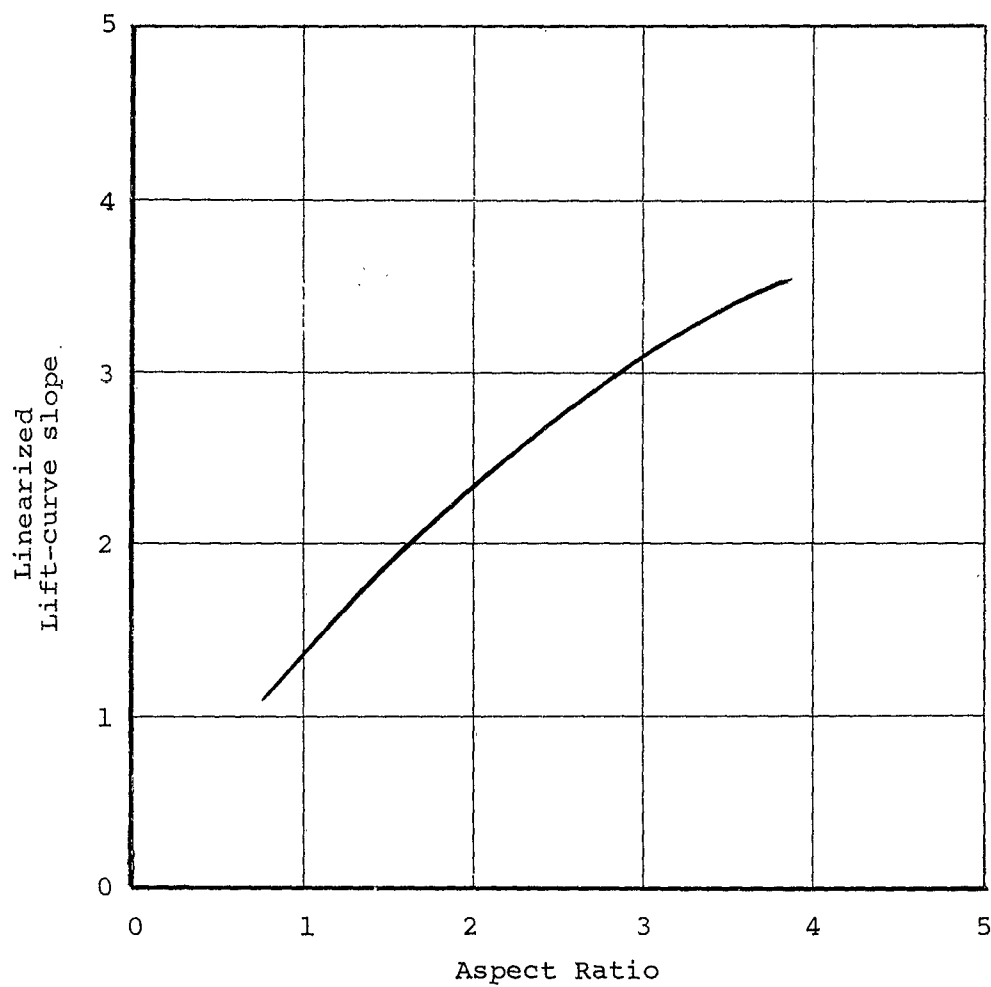


Figure 1.- Subsonic lift-curve slope according to linearized theory (Ref. 17).

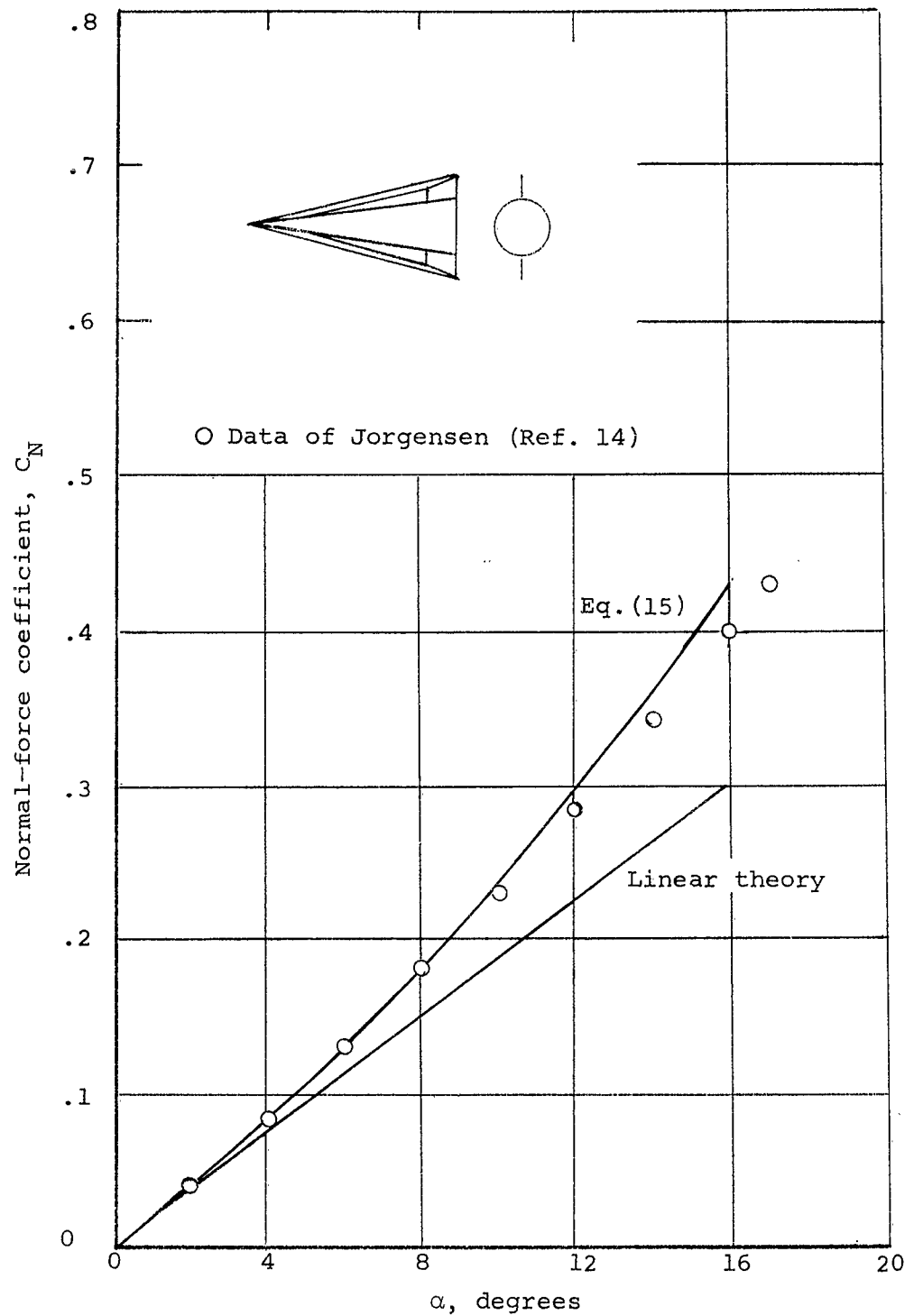


Figure 2.- Theoretical and experimental normal force for slender wing-body combination of aspect ratio 1.0 at  $M_\infty = 1.97$  ( $\frac{a}{b} = 1$ ).

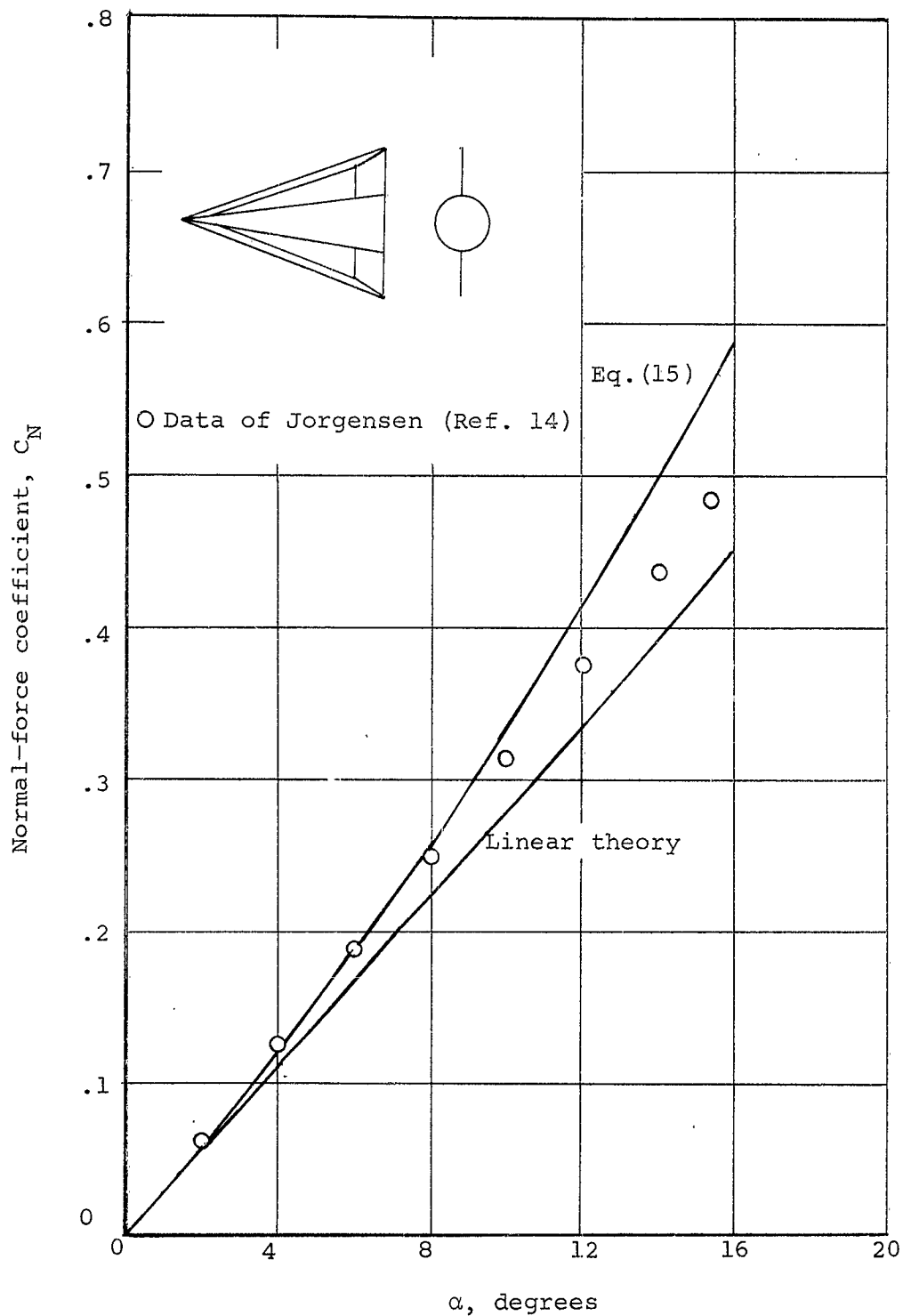


Figure 3.- Theoretical and experimental normal force for slender wing-body combination of aspect ratio 1.5 at  $M_\infty = 1.97$  ( $\frac{a}{b} = 1$ ).

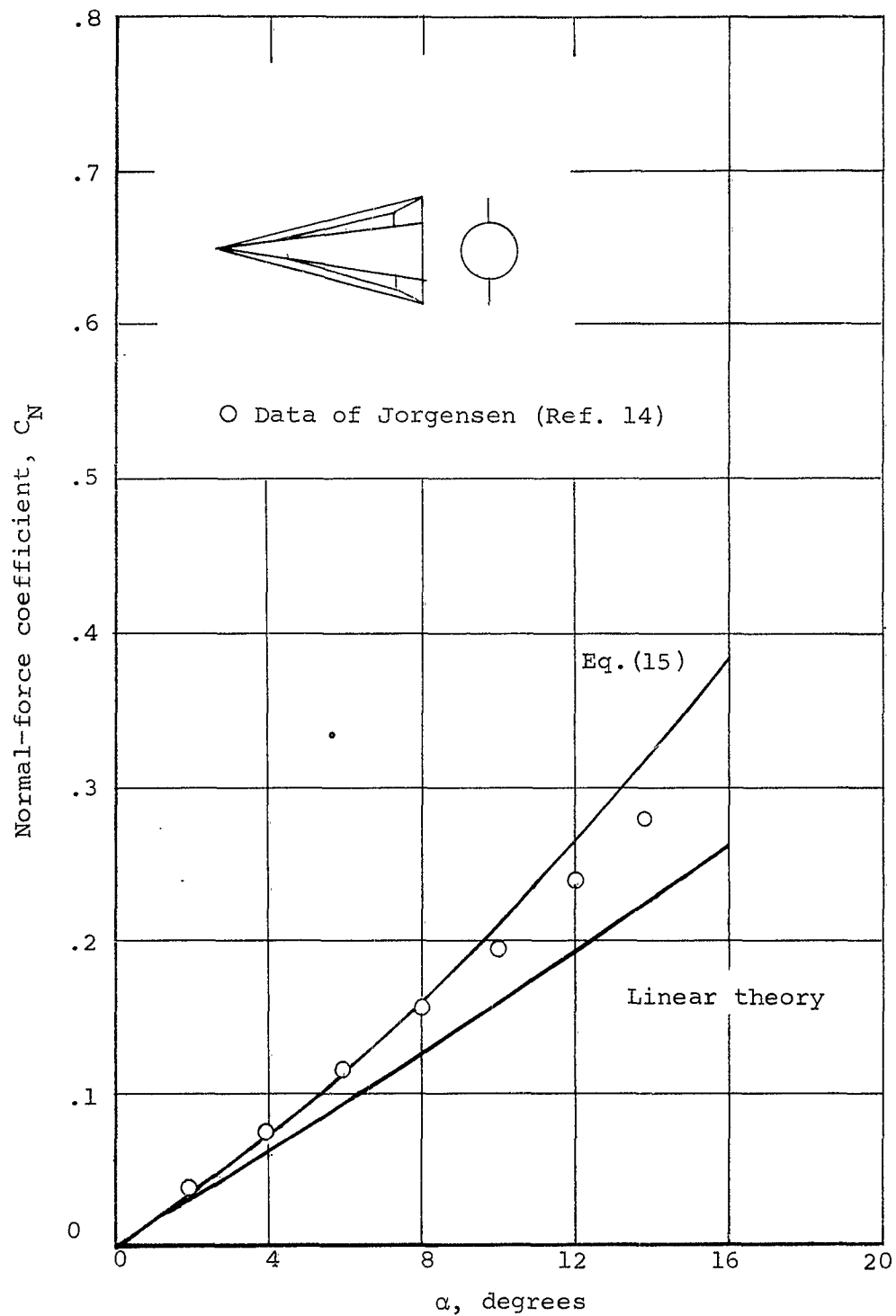


Figure 4. - Theoretical and experimental normal force for slender wing-body combination of aspect ratio 1.0 at  $M_\infty = 2.94$  ( $\frac{a}{b} = 1$ )

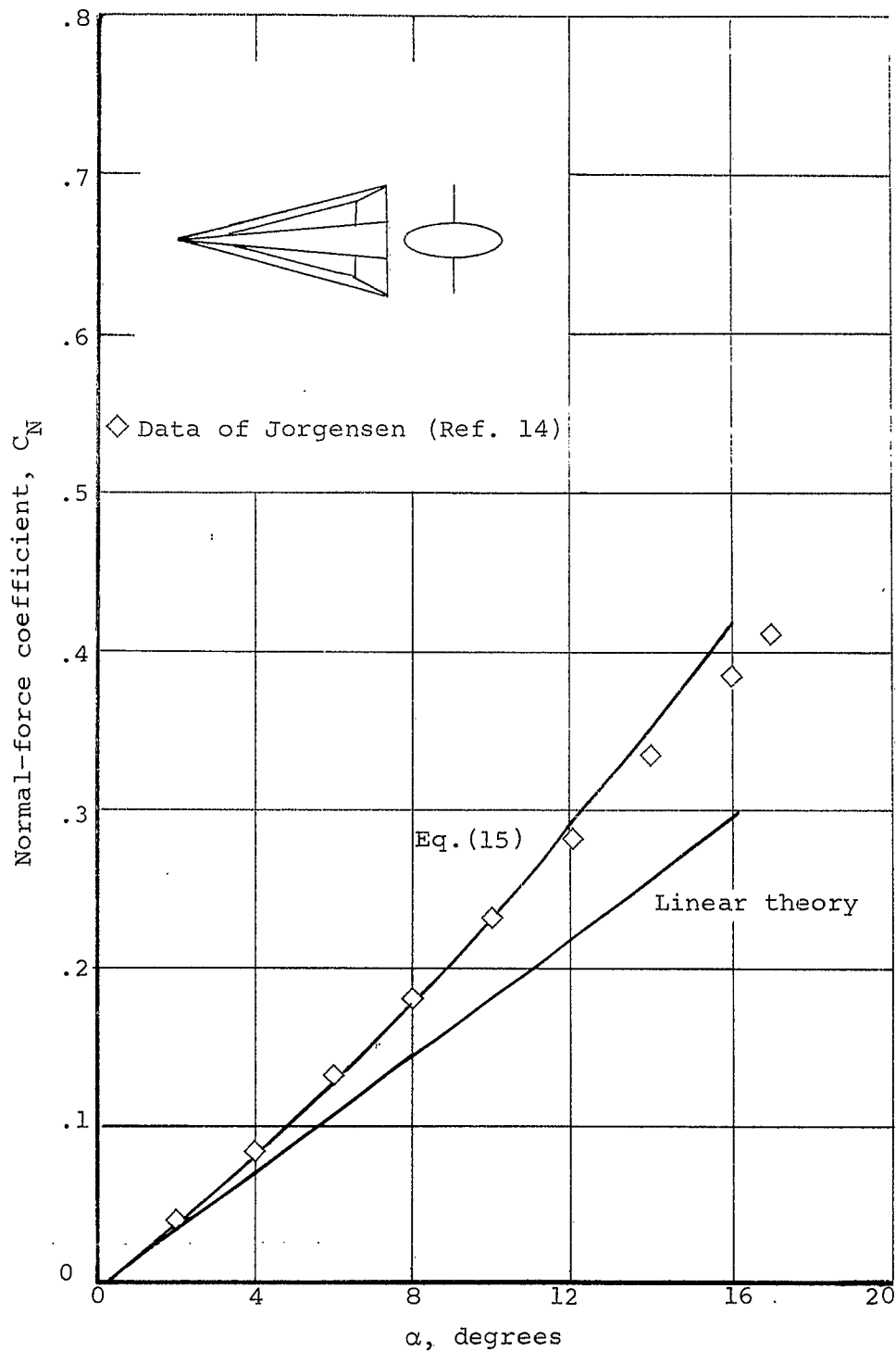


Figure 5. - Theoretical and experimental normal force for slender wing-body combination of aspect ratio 1.0 at  $M_\infty = 1.97$   $\left(\frac{a}{b} = 3\right)$

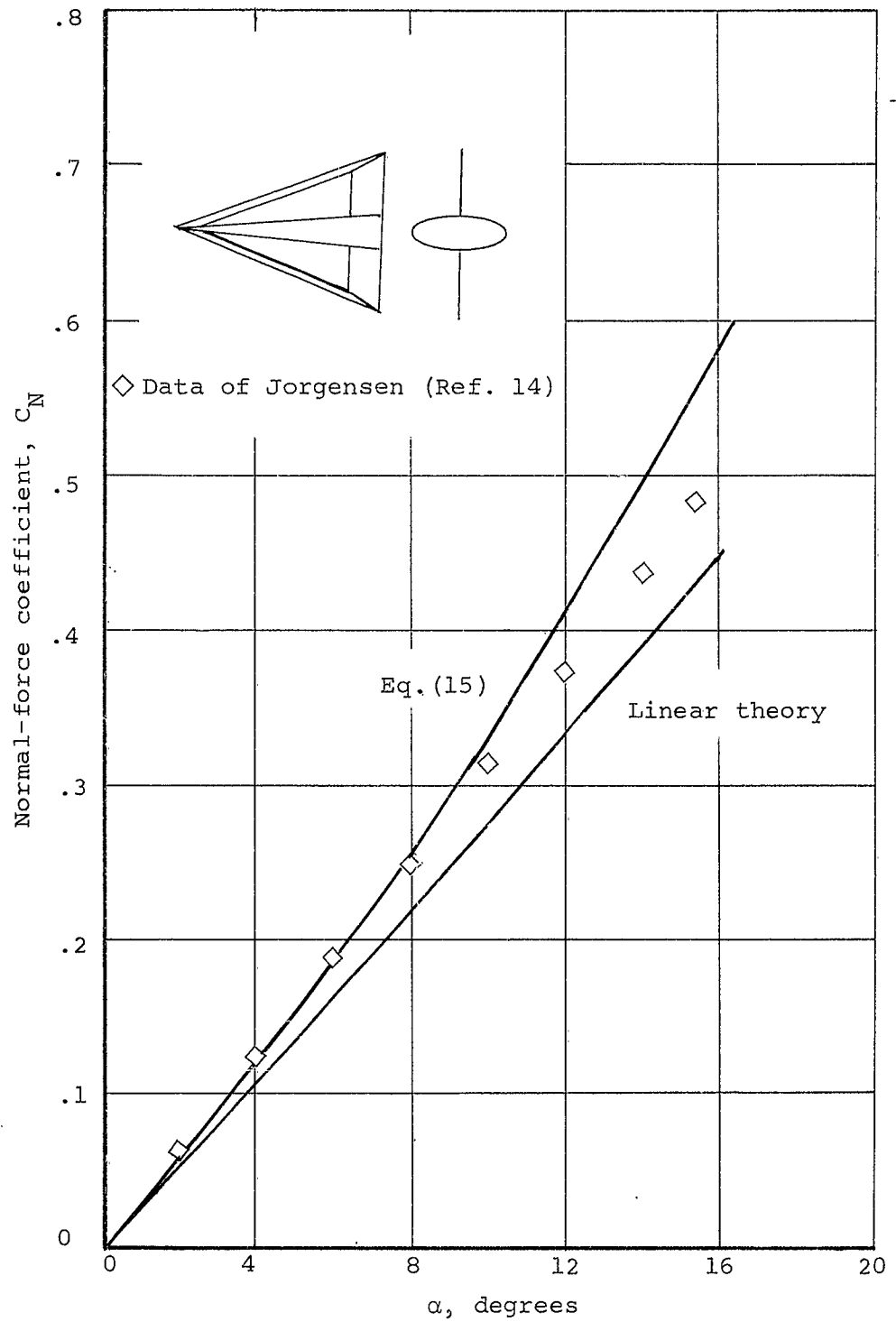


Figure 6. - Theoretical and experimental normal force for slender wing-body combination of aspect ratio 1.5 at  $M_\infty = 1.97$   $\left(\frac{a}{b} = 3\right)$



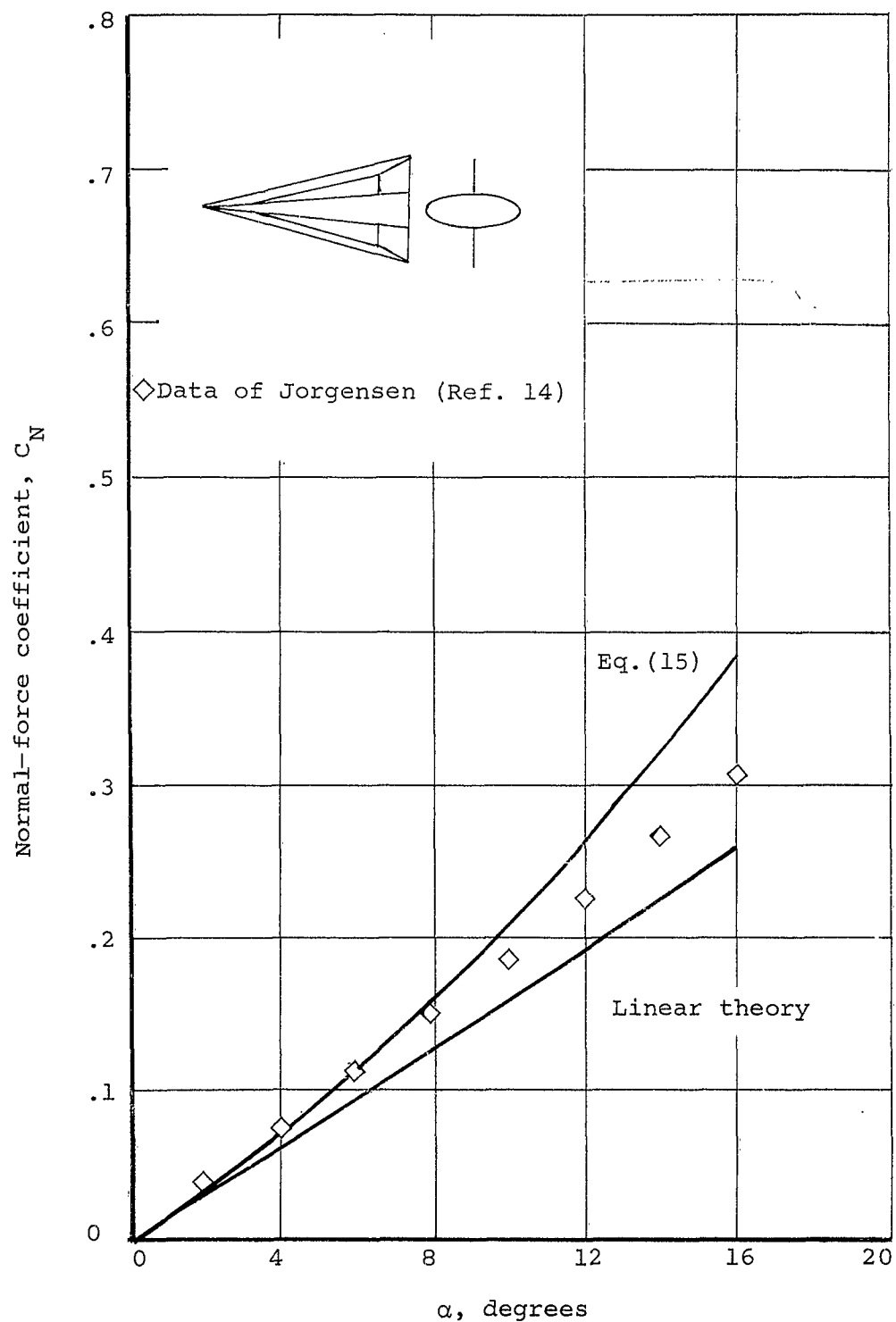


Figure 7. - Theoretical and experimental normal force for slender wing-body combination of aspect ratio 1.0 at  $M_\infty = 2.94$  ( $\frac{a}{b} = 3$ ).

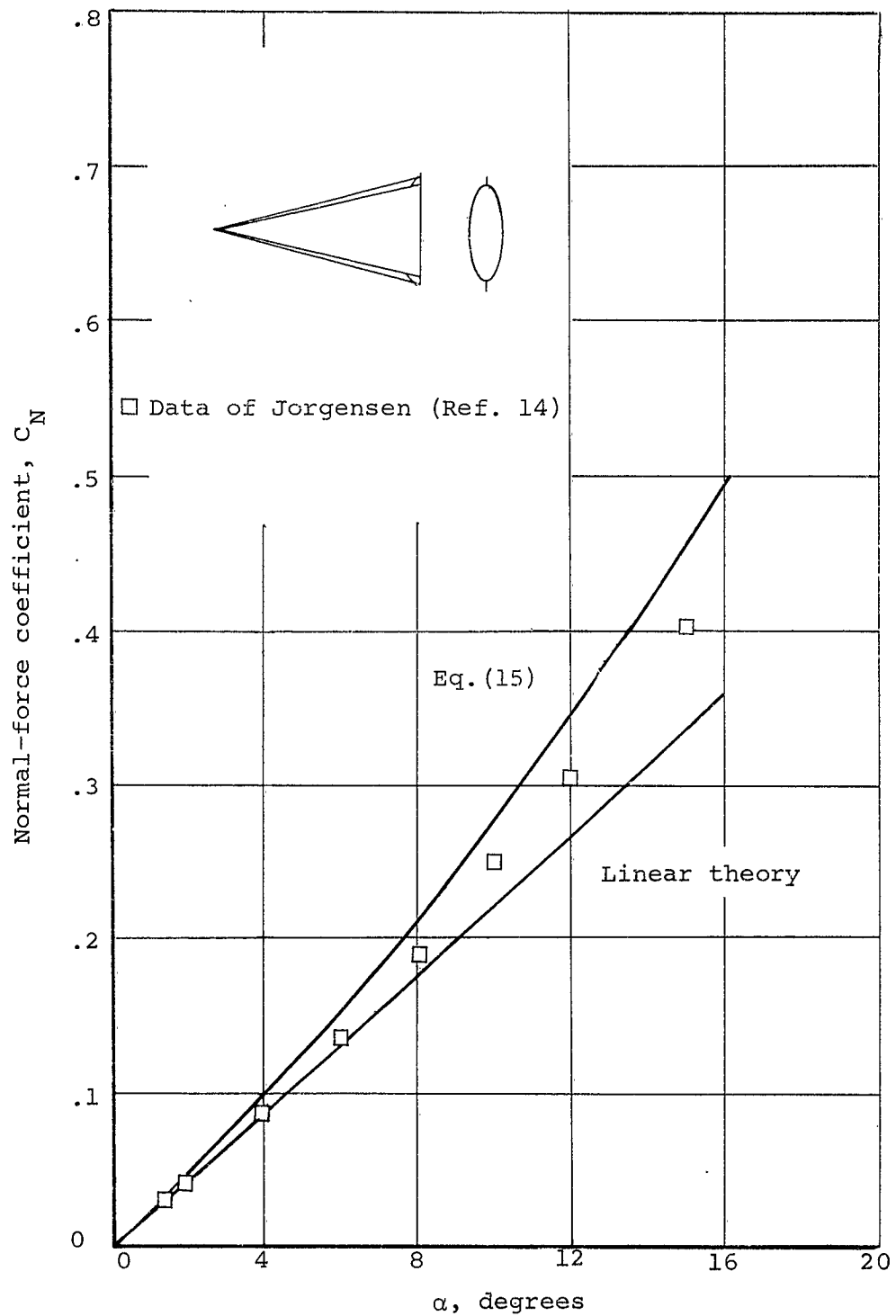


Figure 8. - Theoretical and experimental normal force for slender wing-body combination of aspect ratio 1.0 at  $M_\infty = 1.97$   $\left(\frac{a}{b} = \frac{1}{3}\right)$ .

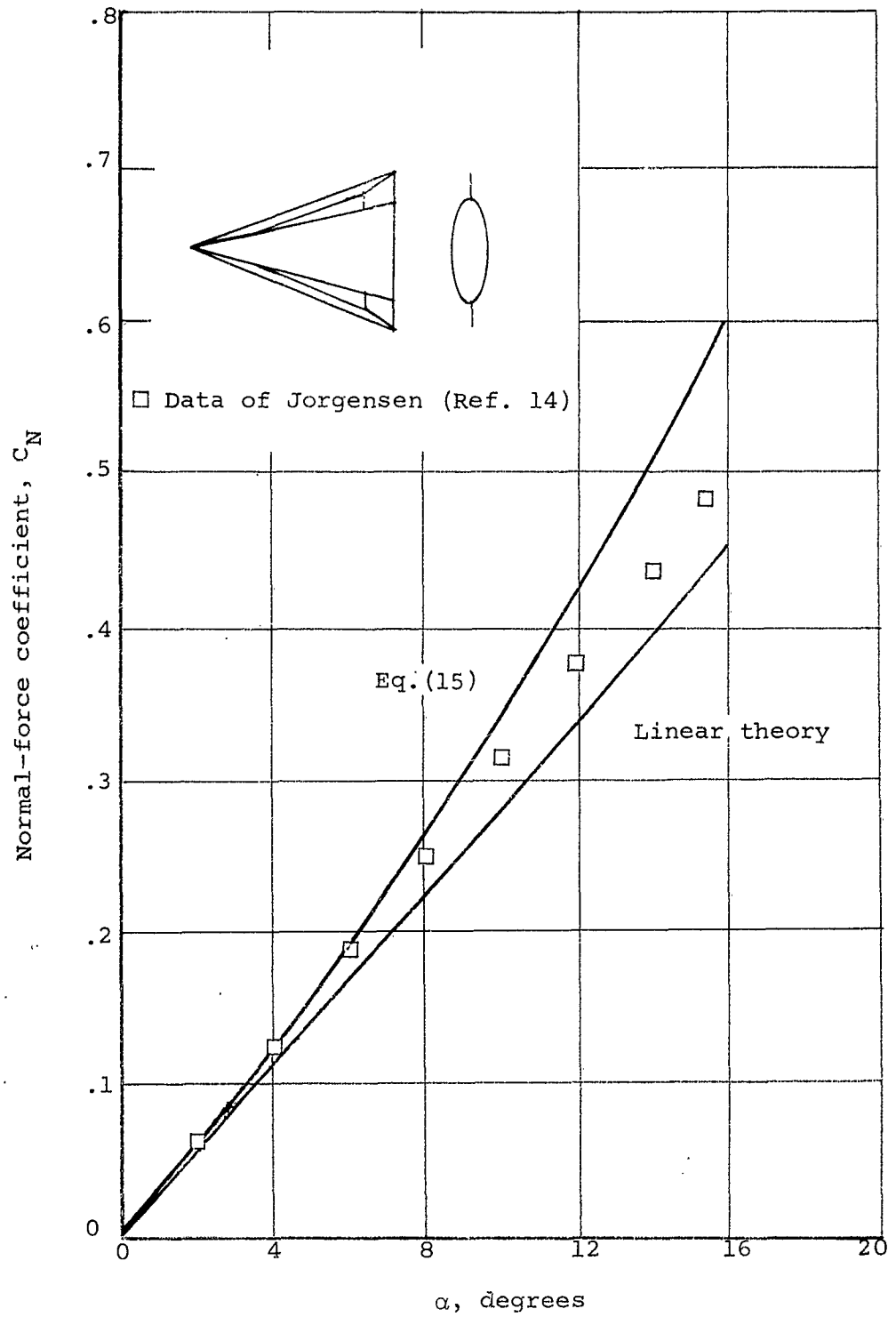


Figure 9.- Theoretical and experimental normal force for slender wing-body combination of aspect ratio 1.5 at  $M_\infty = 1.97$   $\left(\frac{a}{b} = \frac{1}{3}\right)$ .

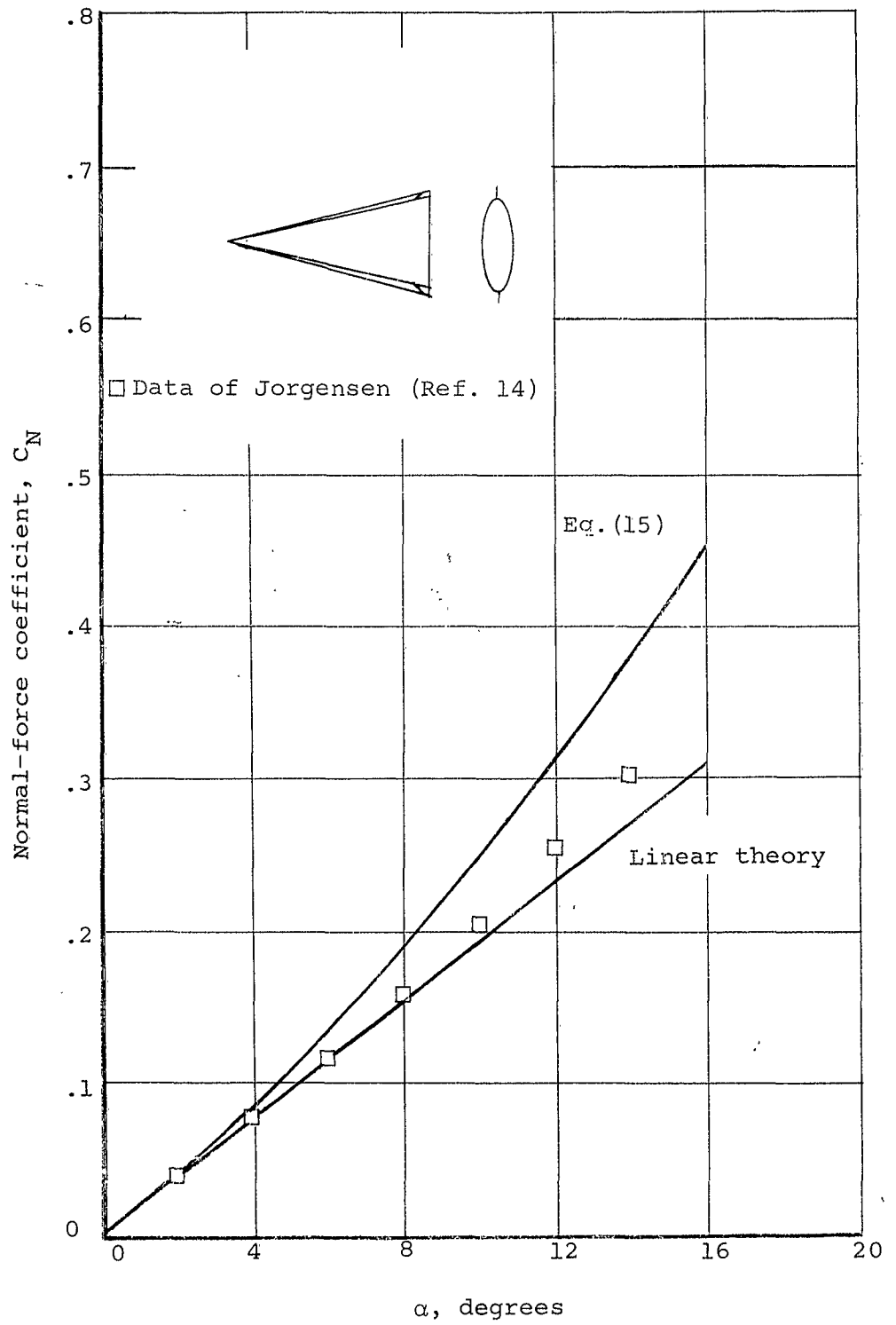
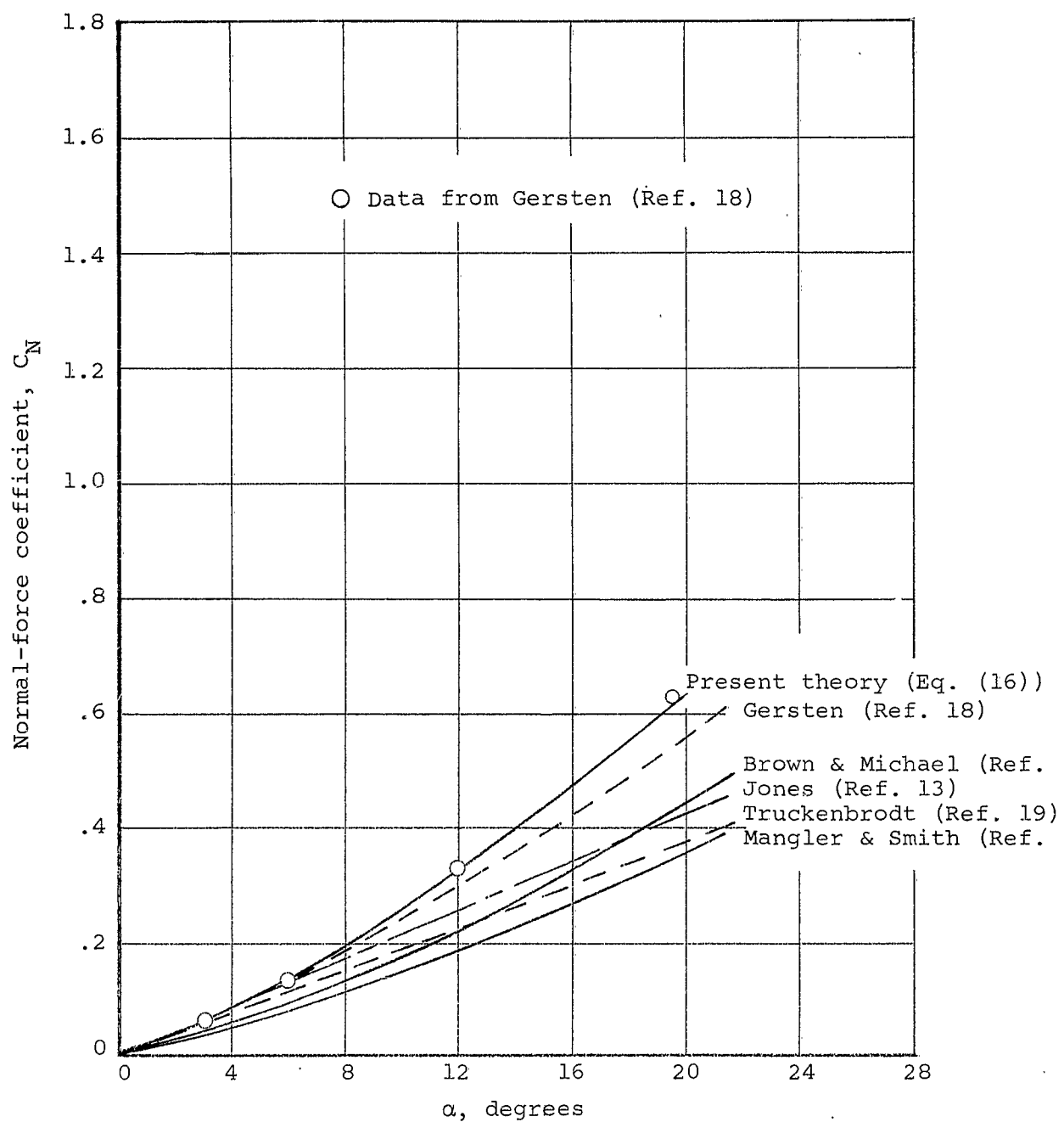
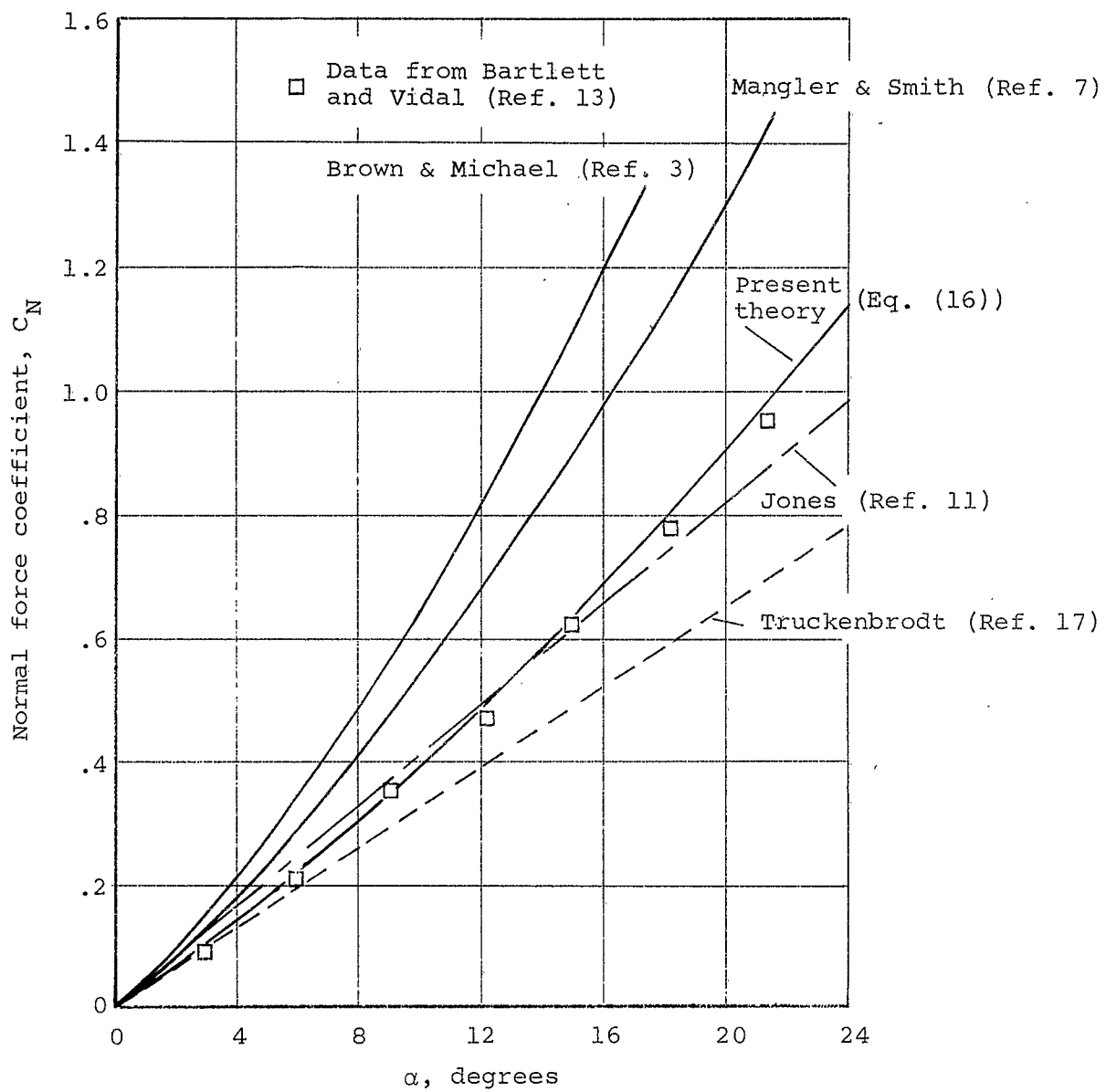


Figure 10.- Theoretical and experimental normal force for slender wing-body combination of aspect ratio 1.0 at  $M_\infty = 2.94$   $\left(\frac{a}{b} = \frac{1}{3}\right)$ .



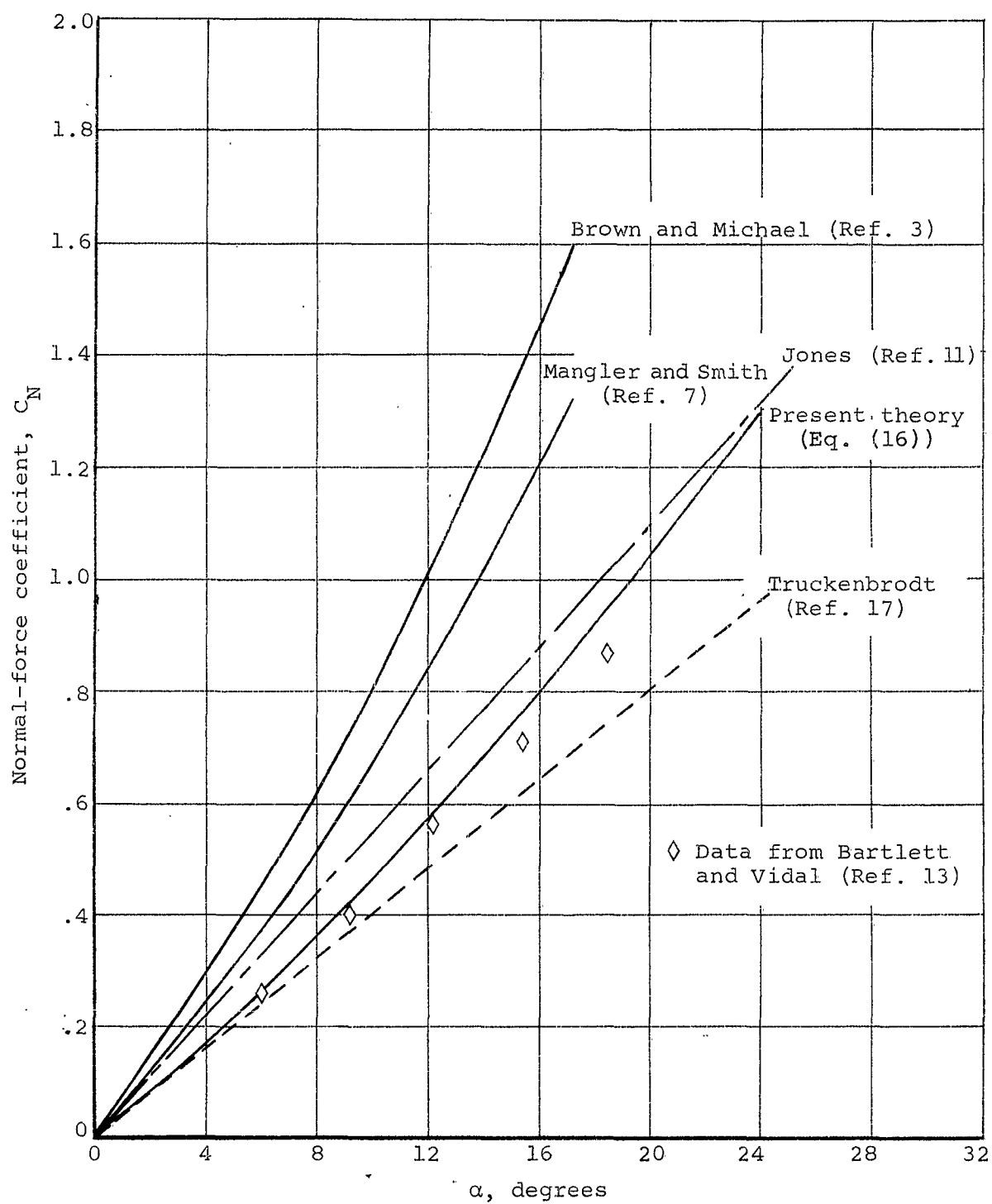
(a)  $A = 0.78$ .

Figure 11.- Theoretical and experimental normal force for flat-plate delta wings.



(b)  $A = 1.5$ .

Figure 11.- Continued.



(c)  $A = 2$

Figure 11.- Concluded.

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